AL-RAZI

ACADEMIC Notes

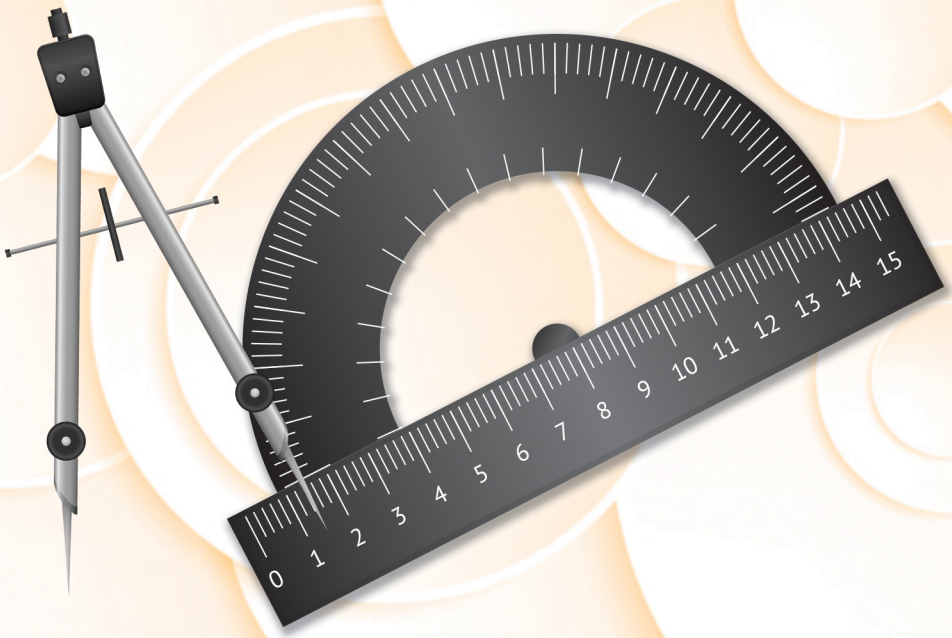
Mathematics Grade 9

Promising:

- Topical MCQs
- Topical Short Questions
- Solved Exercises

All Questions In:

- Try Yourself (Solved)
- Brain Teaser (Solved)
- Think (Solved)



According to Single National Curriculum

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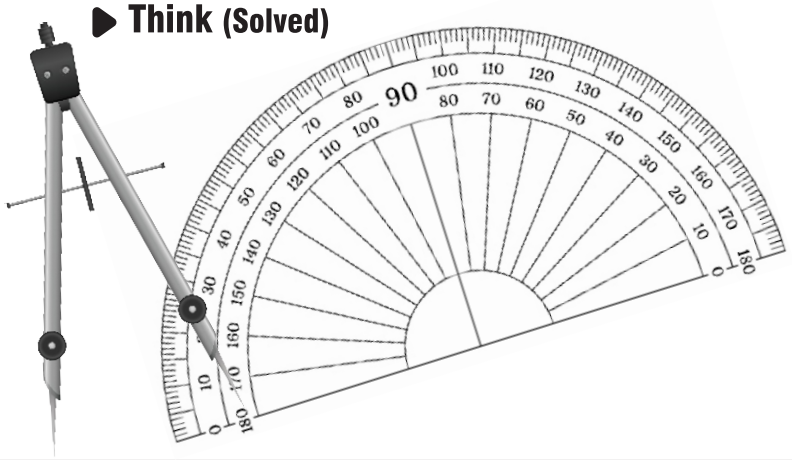

Contents

Preface

Welcome to the Al-Razi Academic Notes Mathematics Grade 9! This comprehensive solution book is designed to support students, teachers, and parents in navigating the exciting world of mathematics.

As students embark on their Grade 9 mathematics journey, they will encounter new concepts, challenge their problem-solving skills, and develop critical thinking. This resource book aims to provide a solid foundation, clarity, and practice opportunities to reinforce learning.

The content within these pages aligns with the Grade 9 mathematics national curriculumand offers:

- Clear explanations and examples
- Varied exercises and practice questions
- Real-world applications and connections
- Opportunities for critical thinking and problem-solving

This resource book is intended to be a companion to your regular textbook and classroom instruction. We hope that it will become a trusted reference, a source of inspiration, and a tool to help you achieve success in mathematics.

We wish you all the best on your mathematical journey!

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Unit 1 Real Numbers

EXERCISE 1.1

Q.1: Identify each of the following as a rational or irrational numbers:

(i) 2.353535

Sol: 2.353535 is a recurring decimal number, therefore it is a **rational number**.

(ii) $0.\overline{6}$

Sol: $0.\overline{6}$ = 0.666 is a recurring decimal number, therefore it is a **rational number**.

(iii) 2.236067…

Sol: 2.236067… is a non-terminating and non-recurring decimal number. Therefore it represents an **irrational number**.

(iv) $\sqrt{7}$

Sol: $\sqrt{7}$ =2.6457... is a non-terminating and non-recurring decimal number. Therefore it represents an **irrational number**.

(v) e

Sol: e = 2.7182818284... is a non-terminating and non-recurring decimal number. Therefore it represents an **irrational number**.

(vi) π

Sol: π = 3.141592653... is a non-terminating and non-recurring decimal number. Therefore it represents an **irrational number**.

Try Yourself!

❍ **What will be the product of two irrational numbers?**

Sol: The product of two irrational numbers can be either rational or irrational. For example:

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$ (irrational)

and $\sqrt{5} \times \sqrt{5} = 5$ (rational)

(vii) $5+\sqrt{11}$

Sol: $5+\sqrt{11}$ = 8.31662479... is a non-terminating and non-recurring decimal number. Therefore it represents an **irrational number**.

(viii) $\sqrt{3}+\sqrt{13}$

Sol: $\sqrt{3}+\sqrt{13}$ = 5.33760208... is a non-terminating and non-recurring decimal number. Therefore it represents an **irrational number**.

(ix) $\frac{15}{4}$

Sol: $\frac{15}{4}$ =3.75 is a terminating decimal number, so it is a **rational number**.

(x) $(2-\sqrt{2})(2+\sqrt{2})$

Sol: $(2-\sqrt{2})(2+\sqrt{2}) = (2)^2 - \left(\sqrt{2}\right)^2$

$= 4 - 2 = 2$

So, the given number is **rational number**.

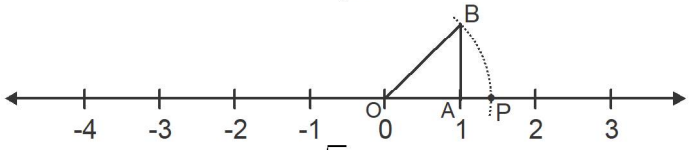
Q.2: Represent the following numbers on number line:

(i) $\sqrt{2}$

Sol: $\sqrt{2}$ can be located on the number line by geometric construction. As $\sqrt{2}$ =1.414... which is near to 1. Draw a line of $m\overline{AB}$=1 unit at point A, where $m\overline{OA}$=1 unit, and we have a right-angled triangle OAB. by using pythagoras theorem.

$m(\overline{OB})^2 = m(\overline{OA})^2 + (m\overline{AB})^2$

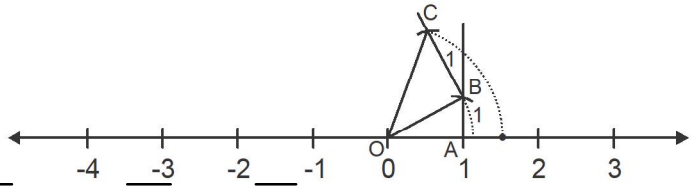
$= (1)^2+(1)^2 = 2 \Rightarrow m\overline{OB} = \sqrt{2}$



Draw an arc of radius $mOB=\sqrt{2}$ taking O as centre, we got point 'P" representing $\sqrt{2}$ on the number line. So, $|OP| = \sqrt{2}$.

(ii) $\sqrt{3}$

Sol: $\sqrt{3}$ can be constructed on the number line by geometric construction. As $\sqrt{3}$ = 1.732... which is near to 2. The number line can be drawn by following method.



$m(\overline{OB})^2 = m(\overline{OA})^2 + m(\overline{AB})^2$

$= (1)^2 + (1)^2 = \sqrt{2}$

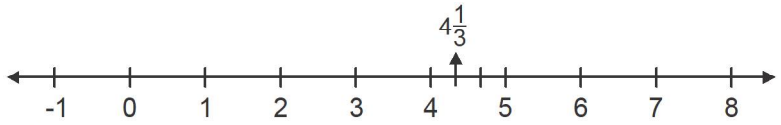
Now $m(OC)^2 = m(\overline{OB})^2 + m(\overline{BC})^2$

$= (\sqrt{2})^2 + (1)^2 = 3 \Rightarrow m\overline{OC} = \sqrt{3}$

(iii) $4\frac{1}{3}$

Sol: $4\frac{1}{3} = 4+\frac{1}{3}$

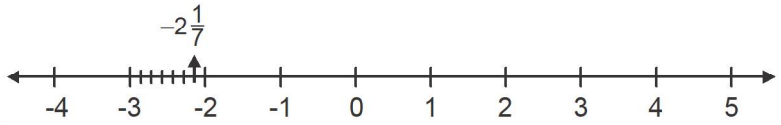
To represent the rational number $4\frac{1}{3}$ on the number line, divide the distance between 4 and 5 into 3 equal parts. Take 1 part to the right.



(iv) $\mathbf{-2\frac{1}{7}}$

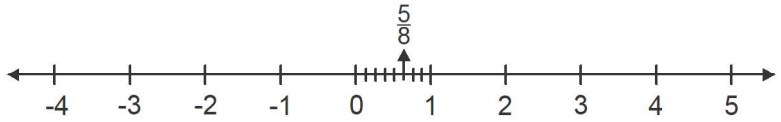
Sol: $-2\frac{1}{7} = -\left(2+\frac{1}{7}\right)$

To represent the rational number $-2\frac{1}{7}$ on the number line, divide the distance between –2 and –3 into 7 equal parts. Take 1 part to the left.



(v) $\mathbf{\frac{5}{8}}$

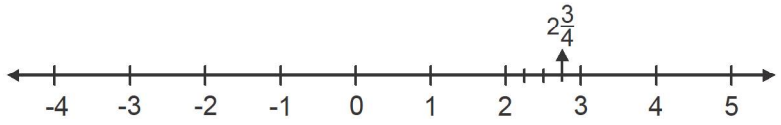
Sol: To represent the rational number $\frac{5}{8}$ on the number line, divide the distance between 0 an 1 into 8 equal parts. Take 5 part to the right.



(vi) $\mathbf{2\frac{3}{4}}$

Sol: $2\frac{3}{4} = 2+\frac{3}{4}$

To represent the rational number $2\frac{3}{4}$ on the number line, divide the distance between 2 and 3 into 4 equal parts. Take 3 part to the right.



Q.3: Express the following as a rational number $\frac{p}{q}$ where p and q are integers and q≠0.

(i) $\mathbf{0.\overline{4}}$

Sol: Let $x = 0.\overline{4}$

$= 0.44444....$ .... (i)

Multiplying both sides by 10.

$10x = 10\ (0.4444....)$

$10x = 4.44444....$ .... (ii)

Subtracting eq. (i) from eq.(ii)

$10x - x = (4.4444....) - (0.4444....)$

$9x = 4$

$\Rightarrow \quad x = \frac{4}{9}$ Which is a rational number.

(ii) $\mathbf{0.\overline{37}}$

Sol: Let $x = 0.\overline{37}$

$= 0.373737....$ .... (i)

Multiplying both sides by 100.

$100x = 100\,(0.373737....)$

$100x = 37.3737....$ .... (ii)

Subtracting eq. (i) from eq.(ii)

$100x - x = (37.3737....) - (0.373737....)$

$99x = 37$

$\Rightarrow \quad x = \frac{37}{99}$ which is a rational number.

(iii) $\mathbf{0.\overline{21}}$

Sol: Let $x = 0.\overline{21}$

$= 0.212121....$ .... (ii)

Multiplying both sides by 100.

$100x = 100(0.212121....)$

$100x = 21.2121....$ .... (ii)

Subtracting eq.(i) from eq.(ii)

$100x - x = (21.2121....) - (0.2121....)$

$99x = 21$

$\Rightarrow \quad x = \frac{21}{99}$ which is a rational number.

Q.4: Name the property used in the following.

(i) $\mathbf{(a+4)+b=a+(4+b)}$

Ans: Associative property over addition.

(ii) $\mathbf{\sqrt{2}+\sqrt{3}=\sqrt{3}+\sqrt{2}}$

Ans: Commutative property over addition.

(iii) $\mathbf{x-x=0}$

Ans: Additive inverse property.

(iv) $\mathbf{a(b+c)=ab+ac}$

Ans: Left distributive property.

(v) $\mathbf{16+0=16}$

Ans: Additive identity property.

(vi) 100×1=100

Ans: Multiplicative identity property.

(vii) 4 × (5 × 8) = (4 × 5) × 8

Ans: Associative property under multiplication.

(viii) ab = ba

Ans: Commutative property under multiplication.

Q.5: Name the properties used in the following:

(i) $-3 < -1 \Rightarrow 0 < 2$

Ans: Additive property.

(ii) If a<b then $\frac{1}{a} > \frac{1}{b}$

Ans: Reciprocal property of inequality.

(iii) If a < b then a + c < b + c

Ans: Additive property of inequality.

(iv) If ac < bc and c > 0 then a < b

Ans: Multiplicative property of inequality.

(v) If ac < bc and c < 0 then a > b

Ans: Multiplication property of inequality.

(vi) Either a > b or a = b or a < b

Ans: Trichotomy property.

Q.6: Insert two rational numbers between:

(i) $\frac{1}{3}$ and $\frac{1}{4}$

Sol: There are infinite rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$. We find any two of them. For this, find the average of $\frac{1}{3}$ and $\frac{1}{4}$ as

$$= \frac{\frac{1}{3}+\frac{1}{4}}{2} = \frac{\frac{4+3}{12}}{2}$$

$$= \frac{7}{12} \times \frac{1}{2} = \frac{7}{24}$$

So, $\frac{7}{24}$ is a rational number between $\frac{1}{3}$ and $\frac{1}{4}$. To find another rational number between $\frac{1}{3}$ and $\frac{1}{4}$, we will again find average of $\frac{7}{24}$ and $\frac{1}{4}$.

i.e; $\frac{\frac{7}{24}+\frac{1}{4}}{2} = \frac{\frac{7+6}{24}}{2} = \frac{\frac{13}{24}}{2} = \frac{13}{48}$

Hence the two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$ are $\frac{7}{24}$ and $\frac{13}{48}$.

(ii) 3 and 4

Sol: There are infinite rational numbers between 3 and 4. We find any two of them. For this, find the average of 3 and 4 as $\frac{3+4}{2} = \frac{7}{2}$. So, $\frac{7}{2}$ is a rational number between 3 and 4. To find another rational number between 3 and 4, we will again find average of $\frac{7}{2}$ and 4. i.e, $\frac{\frac{7}{2}+4}{2} = \frac{\frac{7+8}{2}}{2} = \frac{15}{4}$. Hence two rational numbers between 3 and 4 are $\frac{7}{2}$ and $\frac{15}{4}$.

(iii) $\frac{3}{5}$ and $\frac{4}{5}$

Sol: There are infinite rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$. We find any two of them. For this, find the average of $\frac{3}{5}$ and $\frac{4}{5}$ as

$= \frac{\frac{3}{5}+\frac{4}{5}}{2} = \frac{\frac{3+4}{5}}{2} =$

$= \frac{\frac{7}{5}}{2} = \frac{7}{10}$

So, $\frac{7}{10}$ is a rational number between $\frac{3}{5}$ and $\frac{4}{5}$. To find another rational number between $\frac{3}{5}$ and $\frac{4}{5}$, we will again find average of $\frac{7}{10}$ and $\frac{4}{5}$.

i.e; $\frac{\frac{7}{10}+\frac{4}{5}}{2} = \frac{\frac{7+8}{10}}{2} = \frac{\frac{15}{10}}{2} = \frac{15}{20} = \frac{3}{4}$

Hence two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{7}{10}$ and $\frac{3}{4}$.

☆☆☆☆☆

EXERCISE 1.2

Q.1: Rationalize the denominator of following:

(i) $\dfrac{13}{4+\sqrt{3}}$

Sol:
$$\frac{13}{4+\sqrt{3}} = \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$$
$$= \frac{13(4-\sqrt{3})}{(4)^2-(\sqrt{3})^2} = \frac{52-13\sqrt{3}}{16-3}$$
$$= \frac{52-13\sqrt{3}}{13} = \frac{52}{13} - \frac{13\sqrt{3}}{13}$$
$$= 4-\sqrt{3}$$

(ii) $\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

Sol:
$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{\sqrt{3}(\sqrt{2}+\sqrt{5})}{(\sqrt{3})^2}$$
$$= \frac{\sqrt{6}+\sqrt{15}}{3}$$

(iii) $\dfrac{\sqrt{2}-1}{\sqrt{5}}$

Sol:
$$\frac{\sqrt{2}-1}{\sqrt{5}} = \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{\sqrt{5}\left(\sqrt{2}-1\right)}{\left(\sqrt{5}\right)^2} = \frac{\sqrt{10}-\sqrt{5}}{5}$$

(iv) $\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Sol:
$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}} = \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$$
$$= \frac{\left(6-4\sqrt{2}\right)^2}{(6)^2-\left(4\sqrt{2}\right)^2}$$
$$= \frac{(6)^2+\left(4\sqrt{2}\right)^2-2(6)\left(4\sqrt{2}\right)}{36-32}$$
$$= \frac{36+32-48\sqrt{2}}{4} = \frac{68-48\sqrt{2}}{4}$$
$$= \frac{4(17-12\sqrt{2})}{4} = 17-12\sqrt{2}$$

(v) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Sol:
$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$
$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$
$$= \frac{\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\left(\sqrt{3}\right)\left(\sqrt{2}\right)}{3-2}$$
$$= \frac{3+2-2\sqrt{6}}{1} = 5-2\sqrt{6}$$

(vi) $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Sol:
$$\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$
$$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2-(\sqrt{5})^2}$$
$$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{7-5}$$
$$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2}$$
$$= 2\sqrt{3}(\sqrt{7}-\sqrt{5})$$

Q.2: Simplify the following:

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

Sol: $\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}}$

$= \frac{(16)^{3/4}}{(81)^{3/4}} = \frac{2^{4\times 3/4}}{3^{4\times 3/4}}$

$= \frac{2^3}{3^3} = \frac{8}{27}$

(ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

Sol: $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

$= \left(\frac{4}{3}\right)^{2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

$= \frac{4^2}{3^2} \div \frac{4^3}{9^3} \times \frac{16}{27} = \frac{16}{9} \div \frac{64}{729} \times \frac{16}{27}$

$= \frac{\cancel{16}}{\cancel{9}_{1}} \times \frac{\overset{3}{\cancel{729}}}{\underset{1}{\cancel{64}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{1}{\cancel{27}}} = 12$

(iii) $(0.027)^{-\frac{1}{3}}$

Sol: $(0.027)^{-\frac{1}{3}} = \left(\frac{27}{1000}\right)^{-\frac{1}{3}}$

$= \left(\frac{1000}{27}\right)^{\frac{1}{3}} = \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}}$

$= \frac{10^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}} = \frac{10}{3}$

(iv) $\sqrt[7]{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}z^{7}}}$

Sol: $\sqrt[7]{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}z^{7}}}$

$= \left(\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}\times z^{7}}\right)^{1/7}$

$= \left(x^{14}\times y^{21-14}\times z^{35-7}\right)^{1/7}$

$= \left(x^{14}\times y^{7}\times z^{28}\right)^{1/7}$

$= x^{14\times 1/7}\times y^{7\times 1/7}\times z^{28\times 1/7}$

$= x^2\times y\times z^4 = x^2yz^4$

(v) $\frac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}}$

Sol: $\frac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}} = \frac{5.(5^2)^{n+1} - 5^2(5)^{2n}}{5.(5)^{2n+3} - (5^2)^{n+1}}$

$= \frac{5.(5^{2n+2}) - 5^{2n+2}}{5^{2n+3+1} - 5^{2n+2}} = \frac{5^{2n+2}(5-1)}{5^{2n+2}(5^2-1)}$

$= \frac{4}{25-1} = \frac{4}{24} = \frac{1}{6}$

(vi) $\dfrac{(16)^{x+1}+20(4^{2x})}{2^{x-3}\times 8^{x+2}}$

Sol: $\dfrac{(16)^{x+1}+20(4^{2x})}{2^{x-3}\times 8^{x+2}} = \dfrac{(2^4)^{x+1}+(5\times 2^2)(2^2)^{2x}}{2^{x-3}\times(2^3)^{x+2}}$

$= \dfrac{2^{4(x+1)}+5\times 2^2 2^{2\times 2x}}{2^{x-3}\times 2^{3(x+2)}}$

$= \dfrac{2^{4x+4}+5\times 2^{2+4x}}{2^{x-3}\times 2^{3x+6}} = \dfrac{2^{4x+4}+5\times 2^{4x+2}}{2^{x-3+3x+6}}$

$= \dfrac{2^{4x+4}+5\times 2^{4x+2}}{2^{4x+3}} = \dfrac{2^{4x+2}\times 2^2+5\times 2^{4x+2}}{2^{4x+2}\times 2}$

$= \dfrac{2^{4x+2}(2^2+5)}{2^{4x+2}\times 2} = \dfrac{4+5}{2} = \dfrac{9}{2}$

(vii) $(64)^{-\frac{2}{3}}\div(9)^{-\frac{3}{2}}$

Sol: $(64)^{-\frac{2}{3}}\div(9)^{-\frac{3}{2}} = (4^3)^{-\frac{2}{3}}\div(3^2)^{-\frac{3}{2}}$

$= 4^{3\times\left(-\frac{2}{3}\right)}\div(3^2)^{-\frac{3}{2}}$

$= 4^{-2}\div 3^{-3} = \dfrac{1}{4^2}\div\dfrac{1}{3^3}$

$= \dfrac{1}{16}\div\dfrac{1}{27} = \dfrac{27}{16}$

(viii) $\dfrac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}}$

Sol: $\dfrac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}} = \dfrac{3^n\times(3^2)^{n+1}}{3^{n-1}\times(3^2)^{n-1}} = \dfrac{3^n\times 3^{2n+2}}{3^{n-1}\times 3^{2n-2}}$

$= \dfrac{3^{n+2n+2}}{3^{n-1+2n-2}} = \dfrac{3^{3n+2}}{3^{3n-3}}$

$= \dfrac{3^{3n}\times 3^2}{3^{3n}\times 3^{-3}} = \dfrac{3^2}{3^{-3}}$

$= 3^{2+3} = 3^5 = 243$

(ix) $\dfrac{5^{n+3}-6.5^{n+1}}{9\times5^{n}-4\times5^{n}}$

Sol: $\dfrac{5^{n+3}-6.5^{n+1}}{9\times5^{n}-4\times5^{n}} = \dfrac{5^{n}\times5^{3}-6.5^{n}.5^{1}}{9\times5^{n}-4\times5^{n}}$

$= \dfrac{5^{n}(5^{3}-6\times5)}{5^{n}(9-4)}$

$= \dfrac{125-30}{5} = \dfrac{95}{5} = 19$

Q.3: If $x = 3+\sqrt{8}$ then find the value of:

(i) $x+\dfrac{1}{x}$

Sol: $x+\dfrac{1}{x}$

Given that:

$x = 3+\sqrt{8}$ ....(i)

$\dfrac{1}{x} = \dfrac{1}{3+\sqrt{8}}$

$= \dfrac{1}{3+\sqrt{8}}\times\dfrac{3-\sqrt{8}}{3-\sqrt{8}}$

$= \dfrac{3-\sqrt{8}}{(3)^{2}-(\sqrt{8})^{2}} = \dfrac{3-\sqrt{8}}{9-8}$

$\dfrac{1}{x} = 3-\sqrt{8}$ ....(ii)

Add eq. (i) and (ii)

$x+\dfrac{1}{x} = 3+\sqrt{8}+3-\sqrt{8}$

$x+\dfrac{1}{x} = 6$ ....(iii)

(ii) $x-\dfrac{1}{x}$

Sol: $x-\dfrac{1}{x}$

subtracting eq. (i) and (ii)

$x-\dfrac{1}{x} = 3+\sqrt{8}-(3-\sqrt{8})$

$= 3+\sqrt{8}-3+\sqrt{8}$

$= 2\sqrt{8}$

(iii) $\mathbf{x^2 + \frac{1}{x^2}}$

Sol: $x^2 + \frac{1}{x^2}$

From eq. (iii)

$x + \frac{1}{x} = 6$

Taking square on both sides.

$\left(x + \frac{1}{x}\right)^2 = (6)^2$

$x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 36$

$x^2 + \frac{1}{x^2} + 2 = 36$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 34$

(iv) $\mathbf{x^2 - \frac{1}{x^2}}$

Sol: Since

$x + \frac{1}{x} = 6$

and $\left(x - \frac{1}{x}\right) = 2\sqrt{8}$

We know that

$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= (6)\left(2\sqrt{8}\right)$

$= 12\sqrt{8}$

(v) $\mathbf{x^4 + \frac{1}{x^4}}$

Sol: As $x^2 - \frac{1}{x^2} = 12\sqrt{8}$

Taking square on both sides.

$\left(x^2 - \frac{1}{x^2}\right)^2 = \left(12\sqrt{8}\right)^2$

$(x^2)^2 + \left(\frac{1}{x^2}\right) - 2(x^2)\left(\frac{1}{x^2}\right) = 144(8)$

$x^4 + \frac{1}{x^4} - 2 = 1152$

$x^4 + \frac{1}{x^4} = 1154$

(vi) $\mathbf{\left(x - \frac{1}{x}\right)^2}$

Sol: As $x - \frac{1}{x} = 2\sqrt{8}$

Taking square on both sides

$\left(x - \frac{1}{x}\right)^2 = \left(2\sqrt{8}\right)^2$

$\left(x - \frac{1}{x}\right)^2 = 1152$

Q.4: Find the rational numbers p and q such that $\mathbf{\frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} = p + q\sqrt{2}}$.

Sol: $\frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} = p + q\sqrt{2}$ .... (i)

take $\frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} = \frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} \times \frac{4 - 3\sqrt{2}}{4 - 3\sqrt{2}}$

$= \frac{(8 - 3\sqrt{2})(4 - 3\sqrt{2})}{(4)^2 - (3\sqrt{2})^2} = \frac{8(4 - 3\sqrt{2}) - 3\sqrt{2}(4 - 3\sqrt{2})}{16 - 9(2)}$

$$= \frac{32 - 24\sqrt{2} - 12\sqrt{2} + 9(\sqrt{2})^2}{16 - 18}$$

$$= \frac{32 - 36\sqrt{2} + 18}{-2} = \frac{50 - 36\sqrt{2}}{-2}$$

$$= \frac{50}{-2} - \frac{36\sqrt{2}}{(-2)} = -25 + 18\sqrt{2}$$

So, eq.(i) becomes

$\Rightarrow$ $-25 + 18\sqrt{2} = p + q\sqrt{2}$

Comparing the coefficients of p and q.

$p = -25$, $q = 18$

Q.5: Simplify the following:

(i) $\dfrac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

Sol: $\dfrac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} = \dfrac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}}$

$$= \frac{5^{2\times\frac{3}{2}} \times 3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}} \times 2^{3\times\frac{4}{3}}}$$

$$= \frac{5^3 \times 3^3}{2^5 \times 2^4} = \frac{125 \times 27}{32 \times 16} = \frac{3375}{512}$$

(ii) $\dfrac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

Sol: $\dfrac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

$$= \frac{(2\times3\times3\times3)\times\left((3\times3\times3)^{2x}\right)^{\frac{1}{3}}}{(3\times3)^{x+1} + (2\times2\times2\times3\times3\times3)(3)^{2x-1}} = \frac{2^1\times3^3\times\left((3^3)^{2x}\right)^{\frac{1}{3}}}{(3^2)^{x+1} + (2^3\times3^3)(3)^{2x-1}}$$

$$= \frac{2\times3^3\times(3^{6x})^{\frac{1}{3}}}{3^{2(x+1)} + 2^3\times3^3\times3^{2x-1}} = \frac{2\times3^3\times3^{2x}}{3^{2x+2} + 2^3\times3^{3+2x-1}}$$

$$= \frac{2\times3^{2x+3}}{3^{2x+2} + 2^3\times3^{2x+2}} = \frac{3^{2x+2+1}\times2}{3^{2x+2}(1+2^3)} = \frac{3^{2x+2}\times3^1\times2}{3^{2x+2}(1+8)}$$

$$= \frac{3\times2}{9} = \frac{6}{9} = \frac{2}{3}$$

(iii) $\sqrt{\dfrac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

Sol: $\sqrt{\dfrac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}} = \sqrt{\dfrac{(2^3 \times 3^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-3}{2}}}}$

$= \sqrt{\dfrac{2^{3\times\frac{2}{3}} \times 3^{3\times\frac{2}{3}} \times 5^{2\times\frac{1}{2}}}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}} = \sqrt{\dfrac{2^2 \times 3^2 \times 5}{(25)^{\frac{3}{2}}}}$

$= \sqrt{\dfrac{2^2 \times 3^2 \times 5}{5^{2\times\frac{3}{2}}}} = \left(\dfrac{2^2 \times 3^2 \times 5}{5^3}\right)^{\frac{1}{2}}$

$= \left(\dfrac{2^2 \times 3^2}{5^{3-1}}\right)^{\frac{1}{2}} = \dfrac{2^{2\times\frac{1}{2}} \times 3^{2\times\frac{1}{2}}}{5^{2\times\frac{1}{2}}}$

$= \dfrac{2\times 3}{5} = \dfrac{6}{5}$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

Sol: $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

$= a^{\frac{1}{3}}\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right) + b^{\frac{2}{3}}\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

$= \left(a^{\frac{2}{3}+\frac{1}{3}} - a^{\frac{1}{3}+\frac{1}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{4}{3}}\right) + \left(a^{\frac{2}{3}}b^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}+\frac{2}{3}} + b^{\frac{4}{3}+\frac{2}{3}}\right)$

$= a^{\frac{3}{3}} - a^{\frac{2}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{4}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{4}{3}} + b^{\frac{6}{3}}$

$= a - \cancel{a^{\frac{2}{3}}b^{\frac{2}{3}}} + \cancel{a^{\frac{1}{3}}b^{\frac{4}{3}}} + \cancel{a^{\frac{2}{3}}b^{\frac{2}{3}}} - \cancel{a^{\frac{1}{3}}b^{\frac{4}{3}}} + b^2$

$= a + b^2$

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EXERCISE 1.3

Q.1: The sum of three consecutive integers is forty-two, find the three integers.

Sol: Let a, (a+1) and (a+2) are three consecutive integers, then

$a+(a+1)+(a+2) = 42$

$3a+3 = 42 \Rightarrow 3a = 42-3$

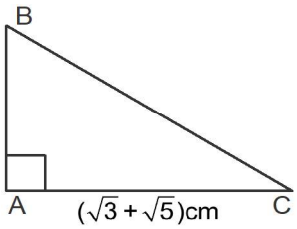
$3a=39 \Rightarrow a = \frac{39}{3} = 13$

So, first integer = a = 13

Second integer = a+1 = 13+1 = 14

Third integer = a+2 = 13+2 = 15

Q.2: The diagram shows right angled ΔABC in which the length of $\overline{AC}$ is $(\sqrt{3}+\sqrt{5})$ cm. The area of ΔABC is $(1+\sqrt{15})$ cm². Find the length $\overline{AB}$ in the form $(a\sqrt{3}+b\sqrt{5})$ cm, where a and b are integers.



Sol: Area of triangle = $(1+\sqrt{15})$ cm²

Length of side (base) = $(\sqrt{3}+\sqrt{5})$ cm

Length of side $\overline{AB}$ = ?

We know that,

Area of triangle = $\frac{1}{2}$ (base × altitude)

$(1+\sqrt{15})\text{cm}^2 = \frac{1}{2}(\overline{AC} \times \overline{AB})$

$2(1+\sqrt{15}) = (\sqrt{3}+\sqrt{5}) \times \overline{AB} \Rightarrow 2+2\sqrt{15} = (\sqrt{3}+\sqrt{5}) \times \overline{AB}$

$$\Rightarrow \overline{AB} = \frac{2+2\sqrt{15}}{\sqrt{3}+\sqrt{5}} = \frac{2+2\sqrt{15}}{\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

$$= \frac{(2+2\sqrt{15})(\sqrt{3}-\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2} = \frac{2(\sqrt{3}-\sqrt{5})+2\sqrt{15}(\sqrt{3}-\sqrt{5})}{3-5}$$

$$= \frac{2\sqrt{3}-2\sqrt{5}+2\sqrt{45}-2\sqrt{75}}{-2}$$

$$= \frac{2\sqrt{3}-2\sqrt{5}+2\times3\sqrt{5}-2\times5\sqrt{3}}{-2} = \frac{2\sqrt{3}-2\sqrt{5}+6\sqrt{5}-10\sqrt{3}}{-2}$$

$$= \frac{-8\sqrt{3}+4\sqrt{5}}{-2} = \frac{-8\sqrt{3}}{-2} + \frac{4\sqrt{5}}{-2}$$

$\overline{AB} = (4\sqrt{3}-2\sqrt{5})$ cm which is required length.

Q.3: A rectangle has sides of length $(2+\sqrt{18})$ m and $(5-\frac{4}{\sqrt{2}})$ m. Express the area of the rectangle in the form a+b $\sqrt{2}$ where a and b are integers.

Sol: Length of rectangle $= (2+\sqrt{18})$ m

Width of rectangle $= \left(5-\frac{4}{\sqrt{2}}\right)$ m

We known that,

Area of rectangle = length × width

$$= (2+\sqrt{18})\left(5-\frac{4}{\sqrt{2}}\right)$$

$$= 10 - \frac{8}{\sqrt{2}} + 5\sqrt{18} - \frac{4\sqrt{18}}{\sqrt{2}}$$

$$= \frac{10\sqrt{2}-8+5\sqrt{36}-4\sqrt{18}}{\sqrt{2}}$$

$$= \frac{10\sqrt{2}-8+5(6)-4\times 3\sqrt{2}}{\sqrt{2}}$$

$$= \frac{10\sqrt{2}-8+30-12\sqrt{2}}{\sqrt{2}} = \frac{22-2\sqrt{2}}{\sqrt{2}}$$

$$= \frac{22}{\sqrt{2}} - \frac{2\sqrt{2}}{\sqrt{2}} = \frac{22\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} - \frac{2\sqrt{2}}{\sqrt{2}}$$

$$= \frac{22\times\sqrt{2}}{2} - 2 \Rightarrow 11\sqrt{2} - 2$$

Area of rectangle $= (-2+11\sqrt{2})m^2$

Which is the required area of the rectangle.

Q.4: Find two numbers whose sum is 68 and difference is 22.

Sol: Let a and b be two required numbers, then

a+b = 68 .... (i)

a–b = 22 .... (ii)

Add eq.(i) and eq.(ii)

$\Rightarrow$ 2a = 90 $\Rightarrow$ a = 45 put in eq.(i)

$\Rightarrow$ 45+b = 68 $\Rightarrow$ b = 68 – 45

b = 23

So, 45 and 23 are required numbers.

Q.5: The weather in Lahore was unusually warm during the summer of 2024. The TV news reported temperature as high as 48°C. By using the formula, $(°F = \frac{9}{5}°C + 32)$ find the temperature as Fahrenheit scale.

Sol: Given that C = 48°C

We know that,

$°F = \frac{9}{5}\ °C+32$

$°F = \frac{9}{5} \times 48+32$

°F = 86.4+32 = 118.4°F

Hence, the temperature in Fahrenheit scale is 118.4°F.

Q.6: The sum of the ages of the father and son is 72 years. Six years ago, the father's age was 2 times the age of the son. What was son's age six years ago?

Sol: Let a and b be the age of father and son respectively, then:

a+b = 72 .... (i)

Six years ago: father's age = a–6 .... (ii)

Son's age = b–6 .... (iii)

By given condition

a – 6 = 2 (b–6)

a – 6 = 2b – 12

a – 2b = –12+6 $\Rightarrow$ a–2b = –6 .... (iv)

Subtract eq. (i) from eq.(iv)

$$\begin{array}{r} a - 2b = -6 \\ \pm\ a \pm\ b = \pm 72 \\ \hline -3b = 78 \end{array}$$

$\Rightarrow \quad b = \frac{-78}{-3}$

$\Rightarrow$ b = 26 put in eq.(iii)

Son's age = 26–6 = 20

So, six years ago, son's age was 20 years.

Q.7: Mirha bought a toy for Rs. 1500 and sold for Rs.1520. What was her profit percentage?

Sol: Cost price of toy = Rs.1500

Selling price of toy = Rs.1520

Profit = S.P. – C.P.

= 1520 – 1500 = Rs.20

Now, we find the profit percentage.

$$\text{Profit\%} = \left(\frac{\text{Profit}}{\text{C.P.}} \times 100\right)\%$$

$$= \left(\frac{20}{1500} \times 100\right)\% = 1.33\%$$

Thus, the profit was 1.33%.

Q.8: The annual income of Tayyab is Rs. 9,60,000, while the exempted amount is Rs. 1,30,000. How much tax would he have to pay at the rate of 0.75%.

Sol: Annual income of Tayyab = Rs. 9,60,000

Exempted amount = Rs. 1,30,000

Taxable amount = Rs. 9,60,000 – Rs. 1,30,000

= Rs. 830,000

Rate of tax = 0.75%

Amount of tax = 830,000 × 0.75%

$$= 830{,}000 \times \frac{0.75}{100} = \text{Rs. } 6{,}225$$

So, Tayyab has to pay Rs. 6,225 as tax.

Q.9: Find the compound markup on Rs. 3,75,000 for one year at the rate of 14% compounded annually.

Sol: Principal amount = P = Rs. 3,75,000

Time = T = 1 year

Rate = R = 14%

Compound Markup = I = ?

To find the compound markup, we have to find the final value after markup (A)

We know that:

$$A = P \times \left(1 + \frac{R}{100}\right)^T$$

$$A = 375000 \times \left(1 + \frac{14}{100}\right)^1$$

$$A = 375000 \times 1.14$$

$$A = \text{Rs. } 427{,}500$$

Now, Compound markup = A – P

= 427500 – 375000

= Rs. 52,500

So, the compound markup is Rs. 52,500.

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REVIEW EXERCISE 1

Q.1: Four options are given against each statement. Encircle the correct option.

(i) $\sqrt{7}$ **is:**

A integer B rational number
c irrational number D natural number

(ii) π **and e are:**

A natural numbers B integers
C rational numbers d irrational numbers

(iii) If n is not a perfect square then $\sqrt{n}$ **is:**

A rational number B natural number
C integer d irrational number

(iv) $\sqrt{3}+\sqrt{5}$ **is:**

A whole number B integer
C rational number d irrational number

(v) For all $x \in R$ **,** $x = x$ **is called:**

a reflexive property B transitive prosperty
C symmetric property D trichotomy property

(vi) Let $a,b,c \in R$ **then** $a>b$ **and** $b>c \Rightarrow a>c$ **is called _____ property.**

A trichotomy b transitive C additive D multiplicative

(vii) $2^x \times 8^x = 64$ **then** $x =$

a $\frac{3}{2}$ B $\frac{3}{4}$ C $\frac{5}{6}$ D $\frac{2}{3}$

(viii) Let a,b, $\in R$ **then a = b and b = a is called _____ property.**

A reflexive b symmetric C transitive D additive

(ix) $\sqrt{75}+\sqrt{27}$ **=**

A $\sqrt{102}$ B $9\sqrt{3}$ C $5\sqrt{3}$ d $8\sqrt{3}$

(x) The product of $(3+\sqrt{5})(3-\sqrt{5})$ **is:**

A prime number B odd number
C irrational number d rational number

2. If $a=\frac{3}{2}$, $b=\frac{5}{3}$ **and** $c=\frac{7}{5}$ **then verify that:**

(i) a(b+c) = ab+ac

Sol: L.H.S. = a(b+c)

put $a=\frac{3}{2}$, $b=\frac{5}{3}$ and $c=\frac{7}{5}$

$$= \frac{3}{2}\left(\frac{5}{3}+\frac{7}{5}\right) = \frac{3}{2}\left(\frac{25+21}{15}\right)$$

$= \frac{3}{2}\left(\frac{46}{15}\right) = \frac{23}{5}$

R.H.S. = ab+ac

put $a=\frac{3}{2}$, $b=\frac{5}{3}$ and $c=\frac{7}{5}$

$= \frac{5}{2}+\frac{21}{10} = \frac{25+21}{10}$

$= \frac{46}{10} = \frac{23}{5}$

Therefore, L.H.S. = R.H.S. Hence proved.

(ii) (a+b)c=ac+bc

Sol: L.H.S.= (a+b)c

put $a = \frac{3}{2}$, $b = \frac{5}{3}$ and $c = \frac{7}{5}$

$= \left(\frac{3}{2}+\frac{5}{3}\right)\times \frac{7}{5} = \frac{3}{2}\times\frac{7}{5}+\frac{5}{3}\times\frac{7}{5}$

$= \frac{21}{10}+\frac{7}{3} = \frac{63+70}{30}$

$= \frac{133}{30}$

R.H.S. = ac+bc

$= \frac{3}{2}\times\frac{7}{5}+\frac{5}{3}\times\frac{7}{5}$

$= \frac{21}{10}+\frac{7}{3} = \frac{63+70}{30}$

$= \frac{133}{30}$

Therefore, L.H.S. = R.H.S. Hence proved.

3. If $a=\frac{4}{3}$, $b=\frac{5}{2}$, $c=\frac{7}{4}$, then verify the Associative property of real numbers w.r.t. addition and multiplication.

Sol: Associative property of real numbers w.r.t addition is:

a+(b+c) = (a+b)+c

L.H.S. = a+(b+c)

put $a=\frac{4}{3}$, $b=\frac{5}{2}$, $c=\frac{7}{4}$

$= \frac{4}{3}+\left(\frac{5}{2}+\frac{7}{4}\right) = \frac{4}{3}+\left(\frac{10+7}{4}\right)$

$= \frac{4}{3}+\frac{17}{4} = \frac{16+51}{12}$

$= \frac{67}{12}$

R.H.S = (a+b)+c

$= \left(\frac{4}{3}+\frac{5}{2}\right)+\frac{7}{4}$

$= \left(\frac{8+15}{6}\right)+\frac{7}{4}$

$= \frac{23}{6} + \frac{7}{4} = \frac{45+21}{12}$

$= \frac{67}{12}$

Therefore, L.H.S. = R.H.S. Hence proved.

Associative property of real numbers w.r.t. multiplication is;

$a(bc) = (ab)c$

L.H.S. $= a(bc)$

put $a = \frac{4}{3}, b = \frac{5}{2}, c = \frac{7}{4}$

$= \frac{4}{3}\left(\frac{5}{2}\times\frac{7}{4}\right) = \frac{4}{3}\times\frac{35}{8}$

$= \frac{35}{6}$

R.H.S. $= (ab)c$

$= \left(\frac{4}{3}\times\frac{5}{2}\right) \times \frac{7}{4}$

$= \frac{10}{3}\times\frac{7}{4}$

$= \frac{35}{6}$

Therefore, L.H.S. = R.H.S. Hence proved.

4. Is 0 a rational number? Explain.

Ans: Yes, 0 is a rational number. We know that a rational number is any number that can be expressed as the ratio of two integers $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

For 0, it can be written as $\frac{0}{1}, \frac{0}{-4}, \frac{0}{100}$ etc. Therefore, 0, meets the definition of a rational number. Hence 0 is a rational number.

5. State trichotomy property of real numbers.

Ans: Trichotomy property states that every real number is either positive, negative or zero.

$\forall a,b \in R$, either $a=b$ or $a>b$ or $a<b$.

6. Find two rational numbers between 4 and 5.

Ans: There are infinite rational numbers between 4 and 5. We find any two of them.

For this, find the average of 4 and 5 as $\frac{4+5}{2}=\frac{9}{2}$ so, $\frac{9}{2}$ is a rational number between 4 and 5. To find another rational number between 4 and 5, we will again find average of $\frac{9}{2}$ and 5.

i.e; $\frac{\frac{9}{2}+5}{2} = \frac{9+10}{2} = \frac{\frac{19}{2}}{2} = \frac{19}{4}$

Hence two rational numbers between 4 and 5 are $\frac{9}{2}$ and $\frac{19}{4}$.

7. Simplify the following:

(i) $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$

Sol: $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} = \left(\frac{x^{15}y^{35}}{z^{20}}\right)^{\frac{1}{5}}$

$= \frac{(x^{15})^{\frac{1}{5}} \times (y^{35})^{\frac{1}{5}}}{(z^{20})^{\frac{1}{5}}} = \frac{x^3y^7}{z^4}$

(ii) $\sqrt[3]{(27)^{2x}}$

Sol: $\sqrt[3]{(27)^{2x}} = \sqrt[3]{(3^3)^{2x}} = \left(3^{3\times 2x}\right)^{\frac{1}{3}}$

$= \left(3^{6x}\right)^{\frac{1}{3}} = 3^{6x\times\frac{1}{3}}$

$= 3^{2x}$

(iii) $\frac{6(3)^{n+2}}{3^{n+1}-3^n}$

Sol: $\frac{6(3)^{n+2}}{3^{n+1}-3^n} = \frac{6\times(3)^n\times 3^2}{3^n\times 3^1-3^n} = \frac{3^n(6\times 3^2)}{3^n(3-1)}$

$= \frac{6\times 9}{2} = 27$

8. The sum of three consecutive odd integers is 51. Find the three integers.

Sol: Let a, a+2, a+4 be three consecutive odd integers.

Then: (a)+(a+2)+(a+4) = 51

3a+6 = 51 $\Rightarrow$ 3a = 51–6

3a = 45 $\Rightarrow$ a = 15

So, first odd integer = a = 15

second odd integer = a+2 = 15+2 = 17

third odd integer = a+4 = 15+4 = 19

9. Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls were in each bucket?

Sol: Total balls = 96

Let number of balls in one bucket = a

Then number of balls in other bucket = a+28

By given condition,

Total balls = (a)+(a+28)

96 = 2a+28 $\Rightarrow$ 2a = 96–28

2a = 68 $\Rightarrow$ a = 34

So, number of balls in one bucket = a = 34

Number of balls in other bucket = a+28 = 34+28 = 62

10. Salma invested Rs. 3,50,000 in a bank, which paid simple profit at a rate of $7\frac{1}{4}\%$ per annum. After 2 years, the rate was increased to 8% per annum. Find the amount she had at the end of 7 years.

Sol: First we will calculate the simple interest for first two years.

Simple interest = $\frac{P \times R \times T}{100}$

Here, P = 3,50,000, R = $7\frac{1}{4}\%$ = 7.25%, T = 2 years

$I_1 = \frac{350000 \times 7.25 \times 2}{100}$ = Rs. 50,750

After 2 years, R = 8% , T = 5 years

$I_2 = \frac{350000 \times 8 \times 5}{100}$ = Rs. 140,000

Total simple interest = 50,750+140,000 = 190,750
Total amount at the end of 7 years will be:
Amount = principal amount + total simple interest
= 350,000+190, 750 = Rs.540,750
So, Salma has to pay Rs. 540,750 at the end of 7 years.

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Multiple Choice Questions (MCQs)

☆ **Four options are given against each statement. Encircle the correct option.**

1. Which civilization used sexagesimal system for mathematics?
A Egyptians b Sumerians C Romans D Indians

2. Roman numerals use _____ letters to represent different numbers.
A 4 B 5 C 6 d 7

3. Who developed the concept of zero(0)?
a Indians B Sumerians C Romans D Egyptians

4. The set $Q=\{\frac{p}{q};p,q\in z\wedge q\neq 0\}$ is called:
A natural numbers B whole numbers
c rational numbers D irrational numbers

5. R = _____
a $Q\cup Q'$ B $Q\cap Q'$ C $Q\cup W$ D $Q\cap W$

6. Which of the following is not a rational number?
A $\frac{1}{4}$ B $\frac{3}{8}$ C 16 d $\sqrt{17}$

7. e = _____?
a 2.7182 B 2.1287 C 1.7182 D 0.7182

8. 0.444... is a:
A terminating decimal number B non-recurring decimal
c recurring decimal number D all of these

9. Commutative property of addition is:
A $a+b\in R$ B $a+0=0+a$ C $a+(-a)=-a+a=0$ d $a+b=b+a$

10. a(bc) = (ab)c is:
a associative property w.r.t multiplication
B associative property w.r.t addition
C closure property w.r.t. addition
D closure property w.r.t. multiplication

11. Multiplicative identity of real numbers is:

A 0 b 1 C –1 D 2

12. $\forall a \in R$, a = a is a property:

A symmetric property

B cancellation property over addition

c Reflexive property D Trichotomy property

13. Transitive property is:

a $\forall a,b,c \in R$; $a>b \wedge b>c \Rightarrow a>c$ B $\forall a,b \in R$; $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$

C $\forall a,b,c \in R$; $a+c = b+c \Rightarrow a=b$ D $\forall a,b \in R$; $a=b \Rightarrow b=a$

14. An irrational radical with rational radicand is called:

A indices b surd C conjugate D polynomial

15. $\left(\frac{\text{profit}}{\text{CP}} \times 100\right)\% = ?$

A loss% B loss c profit% D profit

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Short Questions

☆ **Solve the following Short Questions.**

1. Express $0.\overline{3}$ as the rational number $\frac{p}{q}$.

Sol: Let $x = 0.\overline{3}$

$x = 0.3333...$ ....(i)

Multiplying both sides by 10.

$10x = (0.3333....) \times 10$

$10x = 3.3333....$ ....(ii)

Subtracting (i) from (ii), we have

$10x – x = (3.3333...) – (0.3333...)$

or $9x = 3 \Rightarrow x = \frac{1}{3}$

Hence $0.\overline{3} = \frac{1}{3}$ which is a rational number.

2. Name the property used in the following:

(i) $(2\times3)\times5=2x(3\times5)$ **(ii)** $5+0=5=0+5$

Sol: (i) Associative property w.r.t. multiplication.

(ii) Additive identity.

3. Evaluate: $\sqrt[3]{16x^4y^5}$

Sol: $\sqrt[3]{16x^4y^5} = \sqrt[3]{(2)(8)(x)(x^3)(y^2)(y^3)}$

$= \sqrt[3]{2xy^2(2^3)(x^3)(y^3)}$

$= \sqrt[3]{2xy^2}\ \sqrt[3]{(2^3)(x^3)(y^{3)}}$

$= \sqrt[3]{2xy^2}\ \sqrt[3]{2^3}\ \sqrt[3]{x^3}\ \sqrt[3]{y^3}$

$= 2xy\sqrt[3]{2xy^2}$

4. Evaluate: $(x^3)^2 \div x^{3^2}$

Sol: $(x^3)^2 \div x^{3^2}$

$= x^{3\times2} \div x^{3\times3}$

$= x^6 \div x^9 = \dfrac{x^6}{x^9}$

$= \dfrac{1}{x^{9-6}} = \dfrac{1}{x^3}$

5. If $x=2+\sqrt{3}$, then find the value of $x-\dfrac{1}{x}$.

Sol: $x = 2+\sqrt{3}$

$\dfrac{1}{x} = \dfrac{1}{2+\sqrt{3}}$

$= \dfrac{1}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \dfrac{2-\sqrt{3}}{4-3}$

$\dfrac{1}{x} = 2-\sqrt{3}$

Now,

$x-\dfrac{1}{x} = 2+\sqrt{3}-(2-\sqrt{3})$

$= 2+\sqrt{3}-2+\sqrt{3}$

$x-\dfrac{1}{x} = 2\sqrt{3}$

6. If the simple profit on Rs.5000 for 5 years is Rs.2000, find the rate of profit.

Sol: Principlal amount = Rs.5000

Simple Profit = Rs.2000

Time = 5 years

To find the rate, we use the following formula;

$$\text{Rate} = \frac{\text{amount of profit} \times 100}{\text{time} \times \text{principal}}$$

$$= \frac{2000 \times 100}{5 \times 5000} = 8\%$$

Thus, rate of profit is 8%.

7. Define terminating decimal numbers.

Ans: A decimal number with a finite number of digits after the decimal point is called a terminating decimal number.

8. What do you know about non-repeating decimals?

Ans. Decimal numbers that do not repeat a pattern of digits after the decimal point continue indefinitely without terminating. Non-repeating decimals are also known as irrational numbers.

9. Write cancellation property over addition of real numbers.

Ans. Cancelation property over addition is given below:

$$\forall a,b,c \in R \; ; \; a+c = b+c \Rightarrow a=b$$

10. Evaluate: $\sqrt{25x^{10n}y^{8m}}$

Sol: $\sqrt{25x^{10n}y^{8m}} = \sqrt{5 \times 5\, x^{10n}\, y^{8m}}$

$$= \left(5^2\, x^{10n}\, y^{8m}\right)^{1/2}$$

$$= 5^{2\times\frac{1}{2}} x^{10n\times\frac{1}{2}} \times y^{8m\times\frac{1}{2}}$$

$$= 5x^{5n}\, y^{4m}$$

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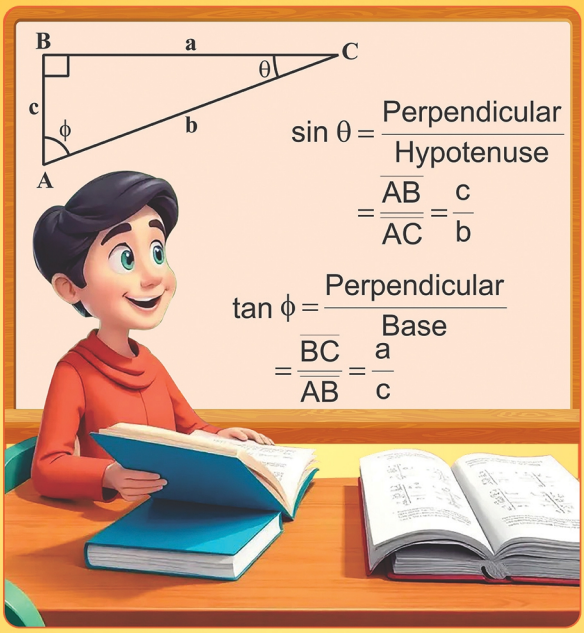

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